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Central Bank Digital Currency, Tax Evasion, and Monetary Policy with Heterogeneous Agents

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Abstract

We investigate the effects of central bank digital currency (CBDC) issuance in an economy where individuals can evade taxes by using cash. Our tractable model features agent heterogeneity with unobservable idiosyncratic shocks and voluntary exchange, where CBDC and cash compete as payment methods. CBDC's transparency enables governments to collect a labor tax that proves non-distortionary in our quasi-linear environment. Agents with higher marginal utility voluntarily pay fixed fees to access interest-bearing CBDC when their debt constraints bind, allowing the implementation of optimal policy with strictly positive inflation and nominal interest rates. We demonstrate how CBDC enables redistribution between agent types that is not possible in cash-only economies. We conjecture that an optimal CBDC policy involves higher nominal interest rates and lower inflation compared to cash regimes. By reducing tax evasion incentives, the introduction of CBDC can increase both output and aggregate welfare.

JEL Classification: E42; E58; H21; H26
Keywords: cash; CBDC; labor tax; tax evasion; monetary policy

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1 Introduction

Tax evasion represents one of the most significant fiscal challenges facing modern economies. The anonymity of cash enables individuals to conceal income and evade taxation: according to Internal Revenue Service estimates, tax evasion costs the United States approximately $441 billion between 2011 and 2013 - roughly 1% of annual GDP (Internal Revenue Service (2019)). As central banks worldwide explore digital currency alternatives, with over 80% now engaged in CBDC-related research and 10% having developed pilot projects (Boar et al. (2020)), a critical question emerges: Can CBDC's transaction transparency reduce tax evasion while maintaining the voluntary exchange that makes money essential? Our paper demonstrates that CBDC can address tax evasion through a mechanism fundamentally unavailable with cash, potentially improving both output and welfare.

This paper's main contribution lies in demonstrating how CBDC can address tax evasion through a transparency mechanism unavailable with cash. The fundamental distinction between the two payment methods lies in observability: cash transactions remain anonymous, allowing agents to hide balances and evade taxes. In contrast, the government can monitor CBDC transactions through centralized technology. This difference creates the possibility for enforceable taxation with CBDC that is impossible with cash alone. We show that when CBDC enables the government to collect a labor tax—which proves non-distortionary in our quasi-linear environment—the introduction of digital currency can generate revenue while simultaneously improving welfare through better insurance against liquidity shocks.

We develop a general equilibrium model extending the frameworks of Xiang (2013) and Andolfatto (2011) - a competitive-market version of the Lagos and Wright (2005) (LW) monetary model without search frictions, similar to the competitive equilibrium version in Rocheteau and Wright (2005). The model incorporates agent heterogeneity in liquidity needs: consumers experience idiosyncratic shocks that create differences in marginal utility, with some agents having higher consumption needs than others. This heterogeneity proves crucial to understanding both the adoption of CBDCs and their redistributive properties. We study optimal monetary policy when only cash, only CBDC, or both payment instruments are available to agents, characterizing equilibria where these methods compete or coexist.

In our framework, CBDC carries both benefits and costs relative to cash. We define CBDC as government-issued money in digital format that can be used for retail transactions through online accounts with the central bank, ensuring widespread accessibility. Agents who use CBDC accept that their transactions become observable to the government, enabling the collection of a labor tax. Additionally, agents can choose between two versions of CBDC: a non-interest-bearing version that functions similarly to digital cash, or an interest-bearing version that pays a nominal return. To access the interest-bearing version, agents must pay a fixed fee after observing their liquidity shock. One can interpret the fixed fee as a redemption or servicing charge that the central bank collects from agents who wish to convert their non-interest-bearing CBDC balances into interest-bearing form. This structure enables low-marginal-utility agents to hold CBDCs without incurring a fee, using them as a simple payment instrument. In contrast, high-marginal-utility agents optimally choose to pay the fee to earn interest on their CBDC holdings.

The inability of the government to implement lump-sum taxation fundamentally shapes our results. In much of the monetary economics literature, governments possess lump-sum tax instruments, allowing optimal policy to follow the Friedman rule: deflation at the rate of time preference financed by lump-sum transfers. In our model, cash anonymity facilitates tax evasion, rendering lump-sum taxation infeasible and necessitating that all trades be voluntary. Without this coercive instrument, optimal policy becomes inflationary, intended to insure individuals against idiosyncratic liquidity shocks while dissuading cash usage that enables evasion. The government can only influence behavior through the relative rates of return on cash and CBDC, adjusted by varying their supplies. To dissuade individuals from using cash, the government must impose positive inflation. If individuals adopt CBDC, inflation persists, but they must be compensated with positive nominal interest rates to maintain voluntary participation.

Our main findings can be summarized as follows. First, the labor tax collected through CBDC is non-distortionary. Although agents using CBDC pay a positive labor income tax, it does not distort equilibrium consumption or production decisions. In this sense, we can view a labor tax as an income tax that raises revenue without efficiency loss. The role of the labor tax in our model is therefore disciplinary — it ensures that agents' behavior remains consistent with efficient outcomes achieved through standard policy instruments, such as inflation. The Friedman rule remains optimal regardless of whether labor income is taxed; however, without lump-sum taxation, it fails to implement the first-best allocation.

Second, we show that tax evasion under cash introduces binding policy constraints. The government's only effective instrument to discourage cash usage is inflation. If a CBDC must offer a higher rate of return than cash to induce adoption, then inflation in the CBDC regime must necessarily be lower than in the cash regime. While we do not provide a formal proof of this claim, our conjecture follows from standard mechanisms widely established in the New Monetarist literature.

Third, our analysis provides insights into optimal CBDC design when tax evasion is a significant concern. We demonstrate that agents with higher marginal utility have the right incentives to adopt interest-bearing CBDC when their debt constraints bind, as they value liquidity most highly and are willing to pay the fixed fee to access the nominal interest rate. Conversely, agents with lower marginal utility may have incentives to misrepresent their types by concealing money balances to acquire higher initial holdings, making them more likely to prefer cash. However, we demonstrate how CBDC implementation can redistribute purchasing power to improve welfare, with the specifics depending on policy parameters. Suppose inflation under CBDC is sufficiently lower than under cash. In that case, even those with lower consumption needs may have the incentive to adopt CBDC despite paying the fixed fee, as the lower inflation provides a higher real rate of return. We derive the equilibrium fixed cost of CBDC that the government can collect and characterize conditions under which redistribution improves social welfare.

1.1 Related Literature

Our paper contributes to the growing body of literature on CBDCs and tax evasion. Specifically, our work closely aligns with Wang (2023), Kwon et al. (2022), and Bajaj and Damodaran (2022). Wang (2023) explores CBDC design while considering tax evasion, portraying agents and the government in a dynamic game where the former is audited by the latter, with inflation dissuading agents from using cash to evade taxes. The author finds that the introduction of an interest-bearing CBDC decreases tax evasion, thereby increasing output and welfare. Kwon et al. (2022) studies tax evasion and CBDC in relation to central bank independence. They introduce a proportional sales tax as a cost associated with CBDC that can potentially lead to distortion. In contrast, the labor tax in our model is non-distortionary and is also tied to the fixed cost of CBDC. Bajaj and Damodaran (2022) examines tax evasion and the informal economy within a Lagos and Wright (2005) framework, where the government expends effort in collecting taxes. They include preference heterogeneity to characterize equilibrium conditions that may arise when agents choose multiple currencies. Unlike our paper, they assume that the government can observe all cash transactions. Other papers that exclusively study tax evasion and the shadow economy within an LW framework include Gomis-Porqueras et al. (2014), Aruoba (2021), and Lahcen (2020).

Several papers examine the welfare implications of CBDC issuance. Williamson (2022) finds that CBDC can reduce crime associated with cash in an environment where banks have limited commitment. The paper posits that CBDC can also economize on the scarcity of safe collateral by paying interest, a point that aligns with our research. Davoodalhosseini (2021) illustrates that CBDC can improve welfare when the central bank can cross-subsidize between different types of agents, a feature not possible with cash. While the author introduces the concept of a nonlinear interest-bearing CBDC, our study illustrates the potential connection between the interest rate on CBDC and its associated costs.

Several papers have examined the impact of CBDC on the banking sector and monetary policy. Assuming a perfectly competitive banking sector, Keister and Sanches (2023) finds that while a CBDC can promote exchange efficiency, it may also increase the funding costs of financial intermediaries and crowd out financial intermediation, thereby preventing an efficient level of investment. Andolfatto (2021) and Chiu et al. (2023) explore the impact of CBDC issuance on banking in imperfectly competitive markets. On the other hand, Whited et al. (2022) uses U.S. bank data to quantify the impact of CBDC on bank lending. They find that if a CBDC pays interest, this may amplify the effect of monetary policy shocks on bank lending.

Issues around financial stability with CBDC issuance have also been studied by several authors, including Brunnermeier and Niepelt (2019), who derive equilibrium conditions in which CBDC can lead to financial stability. Similarly, Kim and Kwon (2019) employs a general equilibrium model of bank liquidity provision, akin to Diamond and Dybvig (1983), to demonstrate that a CBDC does not lead to a credit crunch and thus does not hinder financial stability. More works that tackle this topic include Williamson (2021), Keister and Monnet (2020), and Fernández-Villaverde et al. (2021). In addition to these, several papers have explored the use of multiple payment methods in an LW model, such as Dong and Jiang (2010), Zhu and Hendry (2019), and Chiu and Wong (2015). Lastly, other papers that examine CBDC and monetary policy in a DSGE framework include Barrdear and Kumhof (2021), Ferrari et al. (2022), and Niepelt (2022).

The rest of the paper proceeds as follows. Section 2 describes the environment. Section 3 describes how the government implements its policy rule. Section 4 describes the decision-making problems of the agents. Section 5 characterizes the competitive monetary equilibrium in which cash and CBDC can coexist or exist independently. Section 6 examines the redistributive policy with CBDC in detail. Section 7 concludes.

2 Environment

The model is similar to that of Xiang (2013) and Andolfatto (2011). There is a continuum of infinitely-lived households consisting of consumer-producer pairs, distributed uniformly on the unit interval. Time is discrete and goes forever, indexed by $t = 0, 1, 2, ..., \infty$. In the spirit of Lagos and Wright (2005), each time-period t is divided into two subperiods, labeled *day* and *night*. Households belong to one of two permanent groups: Group 1 and Group 2. Each group is of equal measure. Denote by A and B the sets of Group 1 and Group 2, respectively.

All households meet at a central location during the day. Let $x_t(i) \in \mathbb{R}$ denote consumption (production, if negative) of output during the day by household $i \in A \cup B$ at date t. Linear preferences in $x_t(i)$ imply that utility is transferable. Assuming that the day good is perishable, an aggregate resource constraint implies

$$X \equiv \int_{A \cup B} x_t(i) di \leq 0 \tag{1}$$

for all $t \geq 0$.

Let $\{c_t(i), y_t(i)\} \in \mathbb{R}_+^2$ denote consumption and production, respectively, output at night household $i \in A \cup B$ at date t. The utility from night consumption is given by $\delta_t u(c_t(i))$, where $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$. The utility from night production is given by $g(y_t(i))$, where $g' > 0$ for $y > 0$, $g'' \geq 0$. Following Kocherlakota (2003), we impose a spatial structure for night transactions, labeled *location 1* and *location 2*. After the shock to the consumer type realizes, producers in group 1 (2) households travel to location 2 (1), while consumers in group 1 (2) travel to location 1 (2). This setup implies that a household cannot consume its own output at night. Perishability of the night good implies another aggregate resource constraint

$$\textstyle\int_A c_t(i) di \leq \int_B y_t(i) di \quad \text{and} \quad \int_B c_t(i) di \leq \int_A y_t(i) di. \tag{2}$$

For consumer heterogeneity, we introduce an idiosyncratic shock on consumer types that captures the differences in their marginal utilities. More specifically, at the beginning of each night, consumers experience an idiosyncratic preference shock represented by the parameter $\delta_t(i)$, where $\delta_t(i) \in \left\{\delta^l = 1, \delta^h = \eta\right\}$ and $\eta > 1$. The shock is *i.i.d.* across consumers within each group and across time.

Households discount payoffs across period with the discount factor $\beta \in (0, 1)$, so that their preferences can be represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{x_t(i) + \delta_t(i) u(c_t(i)) - g(y_t(i))\right\}. \tag{3}$$

We focus on symmetric stationary allocations, where all agents are equally weighted and agents of the same type are treated the same, and the two types are treated symmetrically. During the night, the social planner instructs consumers of a representative household to consume $c^j \in \left\{c^l, c^h\right\}$

conditional on type realization. Symmetric locations at night imply that there is a measure 1/4 of type h consumers, a measure 1/4 of type l consumers, and a measure 1/2 of producers; so that the resource constraint (2) can be expressed in another way

$$\frac{1}{4}c^l + \frac{1}{4}c^h = \frac{1}{2}y. \tag{4}$$

The ex-ante lifetime utility of households at a stationary allocation $\left(c^l, c^h, y\right)$ is expressed as

$$W = \frac{1}{(1-\beta)}\left\{\frac{1}{2}\left[u(c^l) + \eta u(c^h)\right] - g(y)\right\}. \tag{5}$$

Linear utility in the day good x implies that any lottery over $\{x_t(i)\}$ for all $t \geq 0$ such that $E_0\left[x_t(i)\right] = 0$ can be a solution. A planner may set $x_t(i) = 0$ for all i households at date $t \geq 0$ as a trivial solution. The first-best allocation $\left(c^{l^*}, c^{h^*}, y^*\right)$ maximizes (5) subject to the aggregate resource constraints (1) and (4).

The first-best allocation is characterized by

$$\begin{aligned} u'(c^{l^*}) &= \eta u'(c^{h^*}), \\ u'(c^{l^*}) &= g'(y^*), \\ c^{l^*} + c^{h^*} &= 2y^*. \end{aligned} \tag{6}$$

In what follows, we impose restrictions on this environment that will make a medium of exchange essential. We assume that households lack commitment and are anonymous. Limited commitment implies that all trades are voluntary, satisfying sequential rationality (individually rational at every period t). Anonymity means that it is impossible to monitor the past actions of agents regarding their trading histories. Given these assumptions, trade by credit is infeasible, so that renders money - in the form of cash and CBDC - essential, as stated by Kocherlakota (1998). Furthermore, we restrict all trades to occur in a sequence of competitive spot markets, where agents exchange cash and CBDC for goods during both day and night. This setup still preserves the essentiality of money even without search frictions as shown in Lagos and Wright (2005).

3 Government Policy

The central bank and the government are a consolidated entity that issues intrinsically worthless tokens called cash and CBDC. Let (M_c, M_e) denote the cash and CBDC supply for the next period, which evolve according to $M_c = \gamma_c M_c^-$ and $M_e = \gamma_e M_e^-$, respectively, where γ_c and γ_e denote the (gross) growth rates of cash and CBDC, respectively (a superscript '-' stands for the cash and CBDC supply of the previous period). The government policy rule is to pay the nominal interest rate R on the CBDC balances to those individuals willing to pay the fixed fee κ for using an interest-bearing CBDC. In addition, the government can also collect taxes τ_x on labor x from each household that uses CBDC as a payment instrument during the day, assuming that a household can hide cash balances to avoid paying the labor tax. In our model, a non-interest-bearing CBDC is equivalent to a digital version of cash; in other words, cash and non–interest–bearing CBDCs are treated the same for policy purposes.

In what follows, the government has an aggregate interest obligation $(R-1)M_e$. The government can also print new money in the form of cash and CBDC, represented as $M_c - M_c^- + M_e - M_e^-$. The government can only collect the fixed CBDC fee $\kappa^j \geq 0$ at night. A household enters the night market and decides whether to pay the fixed fee. If a household pays the fixed fee, then it earns the nominal interest R at night from its CBDC holdings. Both κ^j and R will affect the future CBDC balances carried forward into the next day. If a household declines to pay the fixed fee, then CBDC here works like cash.

We simplify matters by applying a result that holds in this class of quasi-linear models. We design the government policy so that only the mass $1/4$ of agents will voluntarily pay the fixed fee at night and conditional on their initial CBDC balances e_1 (explained in more detail below) will pay the labor tax τ_x during the day. Thus, a feasible government policy will have to satisfy the government budget constraint,

$$(R-1)M_e = M_c - M_c^- + M_e - M_e^- + \tau_x X 1_{e_1>0} + \frac{1}{4}\kappa^j,$$

where $1_{e_1} \geq 0$ is an indicator function, for the labor tax that can be collected only when agents use CBDC as a means of payment, and X is the aggregate labor. Alternatively, we can rearrange the above equation to express the government budget constraint by

$$[(R-2)\gamma_e + 1]\, M_e^- - (\gamma_c - 1)M_c^- = \tau_x X 1_{e_1>0} + \frac{1}{4}\kappa^j. \tag{7}$$

We assume that the cash and CBDC supplies grow at constant rates, with $\gamma_c \geq 1$ and $\gamma_e \geq 1$. For a monetary equilibrium to exist, we require $\gamma_c > \beta$ for the cash economy and $R\beta < \gamma_e$ for the CBDC economy. These conditions ensure that the real return on holding money is strictly dominated by time preference, preventing infinite accumulation of nominal balances. In what follows, we refer to $(R, \gamma_c, \gamma_e, \kappa^j, \tau_x)$ as a government policy. We define a *zero intervention policy* as a policy where $\gamma_c = \gamma_e = R = 1$, so that $\tau_x = \kappa^j = 0$.

4 Decision-Making of Households

During the day, a household decides how much to consume and how much money to carry forward to the night market. At the beginning of the night, the consumer preference shock is realized. Subsequently, the consumer and producer within a household separate and travel to different locations to engage in trade. At that stage, they decide which type of payment instrument to use: cash or CBDC, either in interest-bearing or non-interest-bearing form. Let $\{(v_1, v_2), (w_1, w_2)\}$ denote the price of cash and CBDC in the day and night markets, respectively.

4.1 The Day Market

Households enter the day with $(m_1, e_1) \geq 0$ of nominal balances of cash and CBDC and the night market with $(m_2, e_2) \geq 0$. Households can use cash to trade x_c of the day good and use CBDC to trade x_e. Define $\phi \equiv {}^{v_1}/_{v_2}$ and $\psi \equiv {}^{w_1}/_{w_2}$. Since $x = x_c + x_e$, a household faces the following day-market budget constraint:

$$x = v_1(m_1 - m_2) + \frac{w_1(e_1 - e_2)}{1 + \tau_x}. \tag{8}$$

Denote by $W(m_1, e_1)$ the utility value of a household entering a day with nominal cash and CBDC balances, (m_1, e_1). Also denote by $V(m_2, e_2)$ the utility value of beginning the night with (m_2, e_2) cash and CBDC balances. Note that $V(m_2, e_2)$ denotes the value before a household knows its consumer type. The value functions W and V must satisfy the recursive relationship

$$W(m_1, e_1) \equiv \max_{m_2, e_2 \geq 0} \left\{ v_1(m_1 - m_2) + \frac{w_1(e_1 - e_2)}{1 + \tau_x} + V(m_2, e_2) \right\}. \tag{9}$$

We will later impose assumptions on V so the first-order conditions determine the demand for both cash and CBDC:

$$\frac{\partial V(m_2, e_2)}{\partial m_2} = v_1 \tag{10}$$

and

$$\frac{\partial V(m_2, e_2)}{\partial e_2} = \frac{w_1}{1+\tau_x}. \tag{11}$$

Note that the demand for CBDC decreases with labor tax, τ_x (see also Gomis-Porqueras et al. (2014)). Moreover, all households enter the night with identical cash and CBDC balances $m_2 \in (0, \infty)$ and $e_2 \in (0, \infty)$. In other words, cash and CBDC demand are independent of the initial cash and CBDC holdings (m_1, e_1), which is a standard result in the Lagos-Wright (LW) models. Applying the envelope theorem yields

$$\frac{\partial W(m_1, e_1)}{\partial m_1} = v_1, \tag{12}$$

$$\frac{\partial W(m_1, e_1)}{\partial e_1} = \frac{w_1}{1+\tau_x}. \tag{13}$$

4.2 The Night Market

A household carries over a nominal cash-CBDC portfolio of (m_2, e_2) at night. Consumer preference shocks are realized at the beginning of the night market. Consumers and producers in a household travel to either *location 1* or *location 2*. A household makes the consumption and production decisions, which consumers and producers carry out by traveling to different locations.

Denote by $c^j = c_c^j + c_e^j$ for consumption of a household with realized consumer type j, where c_c^j is type j's consumption by using cash and c_e^j is type j's consumption by using CBDC. Similarly, $y^j = y_c^j + y_e^j$ is the output produced, where a mixture of cash and CBDC can be used for its purchase. Hence, future cash balances are given by

$$m_1^+(j) = m_2 + v_2^{-1}(y_c^j - c_c^j).$$

Future CBDC balances depend on whether a type j consumer chooses to pay the fixed fee κ^j to earn interest on their CBDC balances. A payment of κ^j is required for a type j consumer to receive a nominal interest rate R. The interpretation of κ^j and R is similar to that in Andolfatto (2010), except that in our framework these parameters are introduced in the night market after the realization of consumer types. We adopt this approach because it provides a straightforward way to link shocks to consumer types with the different forms of payment instruments. By contrast, in Andolfatto (2010), there is no consumer heterogeneity and no connection between marginal utilities of consumption and an interest-bearing asset. Our setup also enables us to examine the policy implications for agents' liquidity needs more thoroughly.

Let $\sigma^j \in [0,1]$ represent the probability that a type j consumer pays the fixed fee at date t. If a household of type j opts not to pay the fixed fee, it does not earn any interest on its CBDC holdings in the night market. In this scenario, CBDC functions similarly to cash, except that households pay the labor tax without earning any interest on their CBDC holdings. Hence, the following condition characterizes future CBDC balances[1]

$$e_1^+(j) = \sigma^j \left(Re_2 - \kappa^j\right) + (1-\sigma^j)e_2 + w_2^{-1}\left(y_e^j - c_e^j\right).$$

For a household with realized consumer type $j \in \{l, h\}$, there are two cash and CBDC constraints for consumption of night output; that is,

$$c_c^j \leq v_2 m_2, \tag{14}$$

$$c_e^j \leq w_2\left[\sigma^j\left(Re_2 - \kappa^j\right) + (1-\sigma^j)e_2\right]. \tag{15}$$

Let $\lambda_c^j \geq 0$ and $\lambda_e^j \geq 0$ denote the Lagrange multipliers associated with the constraints (14) and (15), respectively. In what follows, the choice problem at night for a type j household is given by

$$\begin{aligned} V^j(m_2, e_2) \equiv \max_{\substack{\sigma^j, c_c^j, c_e^j,\\ y_c^j, y_e^j}} \Bigg\{ & \delta^j u(c_c^j + c_e^j) - g(y_c^j + y_e^j) \\ & + \beta W\left(m_2 + v_2^{-1}(y_c^j - c_c^j), \sigma^j\left(Re_2 - \kappa^j\right) + (1-\sigma^j)e_2 + w_2^{-1}\left(y_e^j - c_e^j\right)\right) \\ & + \lambda_c^j(v_2 m_2 - c_c^j) + \lambda_e^j\left(w_2\left[\sigma^j\left(Re_2 - \kappa^j\right) + (1-\sigma^j)e_2\right] - c_e^j\right)\Bigg\}. \end{aligned} \tag{16}$$

We now make the following assumptions on the function V:

[1] Note that our CBDC fee structure has a nonlinear mechanism, which could be taken advantage of by a coalition of agents. However, the presumed absence of commitment makes such coalitions infeasible.

Assumption 1 *i)* $V_{m_2,m_2} < 0 < V_{m_2}$ *and* $v_1 < \frac{\partial V(0,e_2)}{\partial m_2}$*;*

ii) $V_{e_2,e_2} < 0 < V_{e_2}$ *and* $\frac{w_1}{1+\tau_x} < \frac{\partial V(m_2,0)}{\partial e_2}$*;*

iii) V *is non-differentiable at* $(R-1)e_2 = \kappa^j$ *for* $\sigma^j \in [0,1]$.

In fact, *Assumption 1* captures the properties of a stationary monetary equilibrium. By applying (12) and (13), all producers regardless of household types, produce identical output y_c with cash and y_e with CBDC satisfying

$$g'(y_c^j + y_e^j) = \beta\frac{\phi}{\gamma_c}, \tag{17}$$

$$g'(y_c^j + y_e^j) = \beta\frac{\psi}{\gamma_e(1+\tau_x)}. \tag{18}$$

The demand for desired consumption c_c^j with cash and the desired consumption c_e^j with CBDC is characterized by the following first-order conditions:

$$\delta^j u'(c_c^j + c_e^j) = \beta\frac{\phi}{\gamma_c} + \lambda_c^j, \tag{19}$$

$$\delta^j u'(c_c^j + c_e^j) = \beta\frac{\psi}{\gamma_e(1+\tau_x)} + \lambda_e^j. \tag{20}$$

If the CBDC constraint binds ($\lambda_e^j > 0$), then a household with realized type j consumer will pay the fixed CBDC fee ($\sigma^j = 1$), as they would like to relinquish their money or discharge their debt for consumption the following day. But if the CBDC constraint is slack ($\lambda_e^j = 0$), then there could be three possibilities, so that the optimal decision to pay the fixed fee κ for a type j consumer satisfies

$$\sigma^j = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \text{if } \kappa^j \begin{cases} < \\ = (R-1)e_2. \\ > \end{cases} \tag{21}$$

As highlighted in Andolfatto (2010), only consumers with sufficiently large money holdings e_2 will find it optimal to pay the fixed fee κ^j given that $R > 1$ and $\kappa^j > 0$. That is, the interest-bearing form of CBDC is used for large-scale transactions, while the cash-equivalent CBDC is used for small-scale transactions. Moreover, by the envelope theorem

$$\frac{\partial V^j(m_2,e_2)}{\partial m_2} = v_2\left(\beta\frac{\phi}{\gamma_c} + \lambda_c^j\right), \tag{22}$$

$$\frac{\partial V^j(m_2, e_2)}{\partial e_2} = \begin{cases} Rw_2 \left[\beta \frac{\psi}{\gamma_e(1+\tau_x)} + \lambda_e^j\right] & \text{if } \kappa^j < (R-1)e_2 \\ w_2 \left[\beta \frac{\psi}{\gamma_e(1+\tau_x)} + \lambda_e^j\right] & \text{if } \kappa^j > (R-1)e_2 \end{cases} \tag{23}$$

We focus on equilibria in which cash and CBDC coexist (or exist independently). This requires that equations (17) and (18) jointly satisfy the following rate-of-return equality condition:

$$\frac{\gamma_e}{\gamma_c} = \frac{\psi}{\phi(1+\tau_x)}. \tag{24}$$

Condition (24) restricts attention to equilibria where cash and CBDC must have the same rate of return from the night to the next day, if they are accepted as payment. Alternatively, condition (24) can also be stated as a no-arbitrage condition. It follows that at the individual level, the cash-CBDC portfolio composition is indeterminate in equilibrium. We define $\zeta = \gamma_e/\gamma_c$ and rewrite this condition as

$$\zeta = \frac{\psi}{\phi(1+\tau_x)}. \tag{25}$$

The above condition ensures the co-existence of cash and CBDC, as they have the same rate of return. If $\zeta > \psi/\phi(1+\tau_x)$, then the rate of return on cash is higher than that of CBDC, so that all individuals will use cash. Conversely, if $\zeta < \psi/\phi(1+\tau_x)$ then all individuals will use CBDC.

Using (25) we can also derive an expression for the labor tax τ_x,

$$\tau_x = \frac{\psi - \zeta\phi}{\zeta\phi}. \tag{26}$$

If agents use only cash, then a tax on labor income is not attainable for the government, so $\tau_x = 0$. If CBDC is used, then $\tau_x \geq 0$. We can obtain an upper bound on τ_x by defining $\overline{\tau_x} \equiv \tau_x < \psi - \zeta\phi/\zeta\phi$. Therefore, the range of values for labor tax is feasible within the interval $[0, \overline{\tau_x}]$, that is, $\tau_x \in [0, \overline{\tau_x}]$. Figure 1 depicts the cases in which cash and CBDC equilibria are separated by the labor tax τ_x. Note that $\zeta = 1$ implies $\phi = \psi$, so that the rate of return on cash and CBDC is exactly equal.

5 Competitive Equilibrium

We now characterize the steady-state equilibria that emerge from the model. Three types of equilibria are possible: economies where only cash is used, economies where only CBDC is used, and economies where households use both payment methods simultaneously.

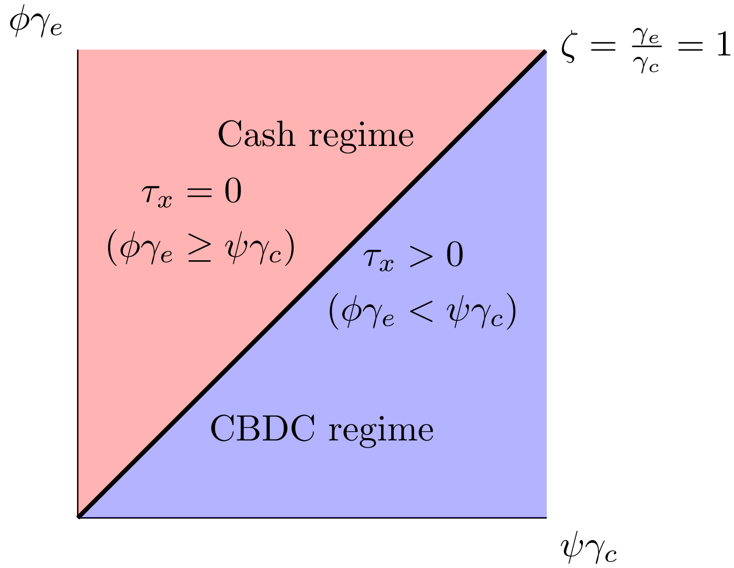

Figure 1: Separation of cash and CBDC equilbria

Before deriving the equilibrium allocations, we establish key assumptions regarding constraint-binding behavior across consumer types. Type h consumers, who experience higher marginal utility from consumption, face more substantial incentives to discharge their debt due to greater liquidity needs. Consequently, both their cash and CBDC constraints are more likely to bind, implying $\lambda_c^h > 0$ and $\lambda_e^h > 0$. Given these binding constraints, type h consumers find it optimal to pay the fixed CBDC fee $\kappa^h = \kappa$ to earn the nominal interest rate R. In contrast, type l consumers have lower liquidity needs due to their lower marginal utility of money. These consumers have incentives to misrepresent their type by concealing money balances to obtain higher initial holdings at the beginning of period 0. As a result, we assume their debt constraints remain slack, that is, $\lambda_c^l = \lambda_e^l = 0$. With slack constraints, type l consumers choose not to pay the fixed CBDC fee, so $\kappa^l = 0$. This asymmetry in fee payment across types creates a natural redistribution mechanism, as type h consumers effectively subsidize CBDC infrastructure that both types may utilize. We later extend this analysis to consider equilibria where type l consumers also face binding constraints, examining how this affects the redistributive properties of CBDC policy.

5.1 CBDC-Only Economy

5.1.1 Market Clearing

Suppose that $\zeta < \psi/\phi(1+\tau_x)$. That is, CBDC offers a higher rate of return than cash, so agents use only CBDC as a means of payment. We have two market-clearing conditions. For the night goods market, equation (4) is the market-clearing condition. The market-clearing condition for the money market involving CBDC is given by

$$e_2 = M_e^-. \tag{27}$$

5.1.2 CBDC Equilibrium

In what follows, we restrict attention to stationary equilibria, which entails $w_1/w_1^+ = w_2/w_2^+ = \gamma_e$. If the CBDC constraint for type h constraint binds then for any $\gamma_e > \beta$, we must have

$$c_e^h = w_2 \left(Re_2 - \kappa \right). \tag{28}$$

First, note that since

$$\frac{\partial V(m_2, e_2)}{\partial e_2} = \frac{1}{2}\frac{\partial V^l(m_2, e_2)}{\partial e_2} + \frac{1}{2}\frac{\partial V^h(m_2, e_2)}{\partial e_2}, \tag{29}$$

combining (23) with $\sigma^l = 0$ and $\sigma^h = 1$ yields

$$\frac{\partial V(m_2, e_2)}{\partial e_2} = \frac{1}{2}\left[w_2 \left[\beta \frac{\psi}{\gamma_e(1+\tau_x)} + \lambda_e^j \right] \right] + \frac{1}{2}\left[Rw_2 \left[\beta \frac{\psi}{\gamma_e(1+\tau_x)} + \lambda_e^j \right] \right]. \tag{30}$$

Combining (20) and (11) leads to

$$\frac{w_1}{1+\tau_x} = \frac{1}{2}u'(c_e^l) + \frac{1}{2}R\eta u'(c_e^h). \tag{31}$$

Now, combining (31) with the market clearing conditions (4) and (27) leads to

$$\left[2\left(\frac{\gamma_e}{R\beta} \right) - \frac{1}{R} \right] u'(c_e^l) = \eta u'(c_e^h). \tag{32}$$

Appealing to (18), one obtains

$$g'(y_e) = u'(c_e^l). \tag{33}$$

The equilibrium allocation $\left(c_e^l, c_e^h, y_e\right)$ for a CBDC economy is then characterized by (32), (33) and (4) when type l consumers are not debt constrained. Note that the "standard" Friedman rule prescription of setting $(R, \gamma_e) = (1, \beta)$ will result in the competitive monetary equilibrium corresponding to the first-best allocation. However, since type l consumers do not willingly pay the fixed fee κ, implementing a deflationary policy according to the Friedman rule is not feasible. Moreover, the labor tax τ_x does not affect the equilibrium allocation, as it is voluntary in the sense that individuals can opt out of paying this tax by using cash instead of CBDC for transactions. We have the following proposition.

Proposition 1 *When $\zeta < \psi/\phi(1+\tau_x)$, in a pure CBDC economy, the labor tax $\tau_x \in (0, \overline{\tau_x}]$ is non-distortionary and does not affect the equilibrium allocation $\left(c_e^l, c_e^h, y_e\right)$.*

5.2 Cash-Only Economy

5.2.1 Market Clearing

Now suppose that $\zeta > \psi/\phi(1+\tau_x)$, so that the rate of return on cash is higher than that of CBDC, agents always demand physical currency and do not accept electronic means of payment. In addition to the night goods market clearing (given by condition (4)), the clearing condition for the money market in the form of cash is

$$m_2 = M_c^-, \tag{34}$$

so that $v_2 = y_c/M_c^-$.

5.2.2 Cash Equilibrium

Similar to the CBDC-only economy, we focus on stationary equilibria in a cash-only economy; in which case $v_1/v_1^+ = v_2^+/v_2^+ = \gamma_c$. Assuming that the cash constraint for type h binds, we will have $c_c^h = v_2 m_2$ for any $\gamma_c > \beta$.

Once again, since

$$\frac{\partial V(m_2, e_2)}{\partial m_2} = \frac{1}{2}\frac{\partial V^l(m_2, e_2)}{\partial m_2} + \frac{1}{2}\frac{\partial V^h(m_2, e_2)}{\partial m_2}, \tag{35}$$

combining (10), (19), and (22) yields

$$\phi = \frac{1}{2}u'(c_c^l) + \frac{1}{2}\eta u'(c_c^h). \tag{36}$$

Combining (19) when type l consumers are not cash constrained with (36) along with the market-clearing conditions (4) and (34) leads to

$$\left[2\left(\frac{\gamma_c}{\beta}\right) - 1\right] u'(c_c^l) = \eta u'(c_c^h). \tag{37}$$

Considering (17), one obtains

$$g'(y_c) = u'(c_c^l). \tag{38}$$

Now, the equilibrium allocation $\left(c_c^l, c_c^h, y_c\right)$ for a cash economy is characterized by (37), (38) and (4). Once again, while the Friedman rule ($\gamma_c = \beta$) could potentially result in the first-best allocation, its implementation is infeasible in this economy due to the lack of a lump-sum tax instrument associated with cash. This limitation arises from the fact that individuals have the option to conceal their cash balances should they desire to consume more of the day good in the following day.

The above equilibrium allocation is assuming that the cash constraint for type l consumers is slack. We now consider when type l consumers are cash constrained, that is, $\lambda_c^l > 0$. Using (14) and the market-clearing condition (4), one obtains

$$c_c^l = c_c^h = y_c. \tag{39}$$

Furthermore, combining (17) and (36) gives rise to

$$g'(y_c) = \frac{\beta}{2\gamma_c}\left(1 + \eta\right) u'(y_c). \tag{40}$$

The equations (39) and (40) fully characterize the equilibrium allocation $\left(c_c^l, c_c^h, y_c\right)$ for a cash economy when the debt-constraints for both type l and type h consumers bind. As the night good y_c and the ex ante welfare W are strictly decreasing in γ_c in both these scenarios, the optimal policy is to set a zero intervention policy with a fixed money supply.

Proposition 2 *When $\zeta > \psi/\phi$ with $\tau_x^* = 0$, in a pure cash economy, the optimal policy is to set $\kappa^* = 0$, which implies $\gamma_c^* = 1$.*

Owing to the limited commitment and anonymity that make lump-sum taxation infeasible, setting the lower limit to $\gamma_c = 1$ optimizes welfare. The second-best solution, as outlined in Xiang (2013), is to maintain a passive policy that minimizes inflation and simultaneously maximizes the rate of return on currency.[2]

6 Redistributive Policy with CBDC

The previous analysis demonstrated that CBDC can eliminate tax evasion and implement non-distortionary labor taxation. However, the introduction of CBDC creates additional possibilities for redistribution between heterogeneous agents that are fundamentally impossible in cash-only economies. This section examines how the transparency mechanism of CBDCs can facilitate redistributive policies by influencing the interaction of fees, interest rates, and differential inflation rates across payment instruments.

The redistributive potential of CBDC emerges from the same transparency feature that prevents tax evasion. Unlike cash transactions, which remain hidden from government observation, CBDC usage is observable, allowing policymakers to design mechanisms that effectively transfer resources between agent types. The key insight is that agents with different marginal utilities will respond differently to the trade-offs between paying CBDC fees and accessing nominal interest rates, creating opportunities for cross-subsidization that preserve individual rationality while improving social welfare.

The mechanism through which this redistribution operates depends on whether type l consumers face binding liquidity constraints and their associated willingness to pay the CBDC fee. When the CBDC constraint for type l consumers remains slack—meaning these consumers retain positive savings and do not pay the fixed fee κ—the government can implement redistribution through a strictly inflationary policy. When type l households have slack debt-constraints, their savings respond negatively to inflation: as the inflation rate rises, the value of their money holdings erodes, effectively reducing their purchasing power. Since inflation operates as an implicit tax on currency holdings, higher inflation disproportionately affects unconstrained type l consumers who maintain positive money balances but do not access interest-bearing CBDC. The government can therefore redirect resources toward type h consumers—who do pay the fee and access the nominal interest rate R—by setting sufficiently high inflation rates that do not reduce type l savings. Such a redistribution channel is feasible only when type l consumers have binding CBDC constraints, which depend on the specific parameter values in the model.

[2]Indeed, this discovery closely aligns with Proposition 1 in Xiang (2013).

6.1 Impatient Economies

To this end, we consider a mixed cash and CBDC economy with a policy of zero intervention; so that $\tau_x = \kappa = 0$ and $\gamma_e^1 = \gamma_c^1 = R = 1$. Using condition (32), the CBDC constraint for both types of consumers will bind when $\beta < 2/(1+\eta)$ conditional on $\eta_0 \equiv \eta > 1$ and $\gamma_e^1 = \gamma_c^1 = 1$ and $R^1 = 1$. As in Andolfatto (2011), we refer to this economy as an impatient economy. Then along with the market-clearing conditions (27) and (34) for the mixed regime, the equilibrium allocation must satisfy (28) and

$$c_e^l = w_2 e_2. \tag{41}$$

Note that first we have to solve for the equilibrium fixed fee, κ, to simplify the equations above. Making use of the government budget constraint (7) and combining with (27), we can obtain

$$\kappa^* = 4\{[(R-2)\gamma_e + 1]M_e^- - (\gamma_c - 1)M_c^- - \tau_x X 1_{e_1>0}. \tag{42}$$

Applying the aggregate resource constraint (1), the latter expression can be further reduced to

$$\kappa^* = 4\{[(R-2)\gamma_e + 1]M_e^- - (\gamma_c - 1)M_c^-\}. \tag{43}$$

Condition (43) gives the optimal κ^* that is necessary to make the debt-constraints bind for both types of consumers. Furthermore, the linear utility in the day good x yields a result that is immediately apparent.

Proposition 3 *κ^* is determined independently of τ_x. Moreover, κ^* is strictly decreasing in γ_c and γ_e when $1 \leq R < 2$, but strictly increasing in γ_e when $R > 2$.*

In other words, given that agents have linear preferences in $x_t(i)$, they are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a specific expected value. As a result, the fixed CBDC fee is not dependent on the labor tax, τ_x. Though the labor tax does not directly generate revenue for the government, it serves as a device to influence individual behavior. In addition, higher CBDC interest rates allow for higher fees only when $R > 2$, while higher cash inflation encourages CBDC adoption with declining fees.

Next, we will derive the equilibrium allocation when agents use CBDC. Using the night goods market-clearing condition (4) along with (28) and (41), this implies

$$\frac{1}{4}(w_2 e_2) + \frac{1}{4} w_2 (R e_2 - \kappa) = \frac{1}{2} y_e;$$

so that

$$e_2 = \frac{1}{1+R}\left[\frac{2y_e}{w_2} + \kappa^*\right]. \tag{44}$$

Now, plugging back everything, the equilibrium allocation is, in this case, given by

$$c_e^l = \frac{1}{1+R}\left[2y_e + w_2\kappa^*\right], \tag{45}$$

$$c_e^h = \frac{1}{1+R}\left[2Ry_e - w_2\kappa^*\right]. \tag{46}$$

From the above conditions, we can verify that type l consumer savings under CBDC, defined as $s_e^l(\gamma_e, \gamma_c, R) \equiv w_2 e_2 - c_e^l(\gamma_e, \gamma_c, R)$, equal zero when the CBDC constraint binds for type l consumers. Similarly, when the cash constraint binds, type l cash savings $s_c^l(\gamma_e, \gamma_c, R) \equiv v_2 m_2(\gamma_e, \gamma_c, R) - c_c^l(\gamma_e, \gamma_c, R)$ also equal zero, as shown in condition (39). This constraint-binding behavior reveals the redistributive mechanism at work. When type h households pay the fixed fee κ to access interest-bearing CBDC, they effectively subsidize the system that both types use. A reduction in the CBDC fee operates as a direct transfer from type l to type h households: lower fees increase the purchasing power available to type h consumers who pay them, while reducing the implicit subsidy that type l consumers receive from the equilibrium price effects. Conversely, higher fees combined with appropriate inflation policies can redirect purchasing power toward type l consumers. The fixed fee and inflation thus serve as purely redistributive instruments, reallocating resources between consumer types.

6.2 Patient Economies

We now consider an economy where agents are sufficiently patient, that is, $\beta \geq 2/(1+\eta)$. In this scenario, with sufficiently low inflation and high nominal interest rates, the cash and CBDC constraints for the l types remain slack. Under these conditions, the equilibrium allocation in a mixed cash-CBDC regime is characterized by (32) and (37). Inflation hurts efficiency as long as both the cash and CBDC constraints remain slack. It is easy to verify that $s_c^l(\gamma_e, \gamma_c, R) \equiv v_2 m_2(\gamma_e, \gamma_c, R) - c_c^l(\gamma_e, \gamma_c, R)$ is monotonically decreasing in γ_c and that $s_c^l(\gamma_e, \gamma_c, R) = 0$ for some $\gamma_c^0 \geq \gamma_c^1$. Considering type l saving with CBDC, we have

$$s_e^l(\gamma_e, \gamma_c, R) = \frac{2y_e + w_2\kappa(\gamma_e, \gamma_c, R) - (1+R)c_e^l(\gamma_e, \gamma_c, R)}{R}. \tag{47}$$

In patient economies where $\beta > 2/1{+}\eta$, type l consumers maintain positive savings under the zero intervention policy: $s_e^l(1,1,1) = 2[y_e - c_e^l(1,1,1)] > 0$. The government's ability to implement redistribution through inflation policy depends on how type l savings respond to the policy parameters (γ_e, γ_c, R).

Type l CBDC savings $s_e^l(\gamma_e, \gamma_c, R)$ respond to policy changes through two distinct channels. The direct channel operates through the fixed fee: since s_e^l is monotonically increasing in κ, higher fees increase savings by tightening the budget constraint. The indirect channel operates through consumption responses: from condition (32), we know that $c_e^l(\gamma_e, \gamma_c, R)$ is monotonically increasing in γ_e and monotonically decreasing in R. Higher CBDC inflation or lower nominal interest rates thus increase type l consumption, reducing their savings through this indirect channel.

The net effect is ambiguous because these channels oppose each other. The optimal CBDC fee responds to γ_e depending on whether $R > 2$, and to R depending on γ_e. If κ were decreasing in γ_e, increasing in γ_c, and increasing in R, then both channels would reinforce: s_e^l would decrease in γ_e, increase in γ_c, and increase in R. Under such conditions, threshold parameters $(\gamma_e^0, \gamma_c^0, R^0) > 1$ would exist such that $s_e^l(\gamma_e^0, \gamma_c^0, R^0) = 0$. The optimal policy would set CBDC inflation and interest rates high enough to make type l consumers debt-constrained, extracting resources for redistribution, yet not so extreme as to discourage accumulation of nominal balances m_2 and e_2. However, without clear signs of $kappa$'s response to policy parameters, we state our result as a conjecture.

Conjecture 1 *When $\beta < 2/(1{+}2\eta)$, the optimal policy is characterized by $\left(\gamma_e^*, \gamma_c^*, R^*\right) > 1$ that satisfies (39), (40), (43), (45), and (46) in a mixed cash-CBDC economy with the condition $\psi \geq \phi\zeta$. When $\beta \geq 2/(1{+}2\eta)$, the optimal policy must satisfy $\gamma_c^* \geq \gamma_c^0$, $\gamma_e^* \geq \gamma_e^0$ and a nominal interest rate $R^* \geq R^0$ so that $s_e^l(\gamma_e^1, \gamma_c^1, R^1) \geq s_c^l(\gamma_e^1, \gamma_c^1, R^1)$, with an equilibrium allocation characterized by (32), (33), (37), and (38). For both these conditions, $\tau_x \in [0, \overline{\tau_x}]$ as defined by (26).*

When $\beta < 2/1{+}2\eta$ and both types are constrained, type h consumers who pay fee κ gain purchasing power. Assuming $\psi \geq \phi\zeta$ ensures CBDC and cash coexist with equal returns, type h consumers use CBDC while type l use cash. The policy question becomes: how to induce type l to switch to CBDC? When $\beta \geq 2/1{+}2\eta$, type l consumers switch if CBDC savings exceed cash savings: $s_e^l(\gamma_e^1, \gamma_c^1, R^1) \geq s_c^l(\gamma_e^1, \gamma_c^1, R^1)$.

A binding CBDC constraint for type l households generates positive government revenue from CBDC fees and labor taxes. Since the labor tax in a CBDC economy is non-distortionary, the introduction of a CBDC may improve welfare. The reason is that, under a cash regime, labor taxation is infeasible as agents can conceal their money holdings to evade taxes. If the CBDC is required to offer a higher rate of return than cash, its inflation rate must therefore be lower. Given that $\beta \geq 2/(1+\eta)$, a lower inflation rate with CBDC relative to cash ($\gamma_e^0 \leq \gamma_c^0$) can increase savings for type l households when they use CBDC instead of cash. By potentially achieving a higher equilibrium level of output y, a CBDC economy can thus improve welfare relative to a cash economy.

6.3 Numerical Example

To analyze the welfare implications of introducing a CBDC relative to a cash-based economy, we conduct a numerical exercise that satisfies the equilibrium restrictions derived earlier. Recall that for a monetary equilibrium to exist, $\gamma_c > \beta$ must hold in the cash economy and $R\beta < \gamma_e$ in the CBDC economy. We fix the parameters at $\beta = 0.96$, $\eta = 1.05$, and $\omega = 0.35$ to ensure these conditions are satisfied.

In the analysis, welfare outcomes for the cash and CBDC economies are computed by solving for the equilibrium consumption levels and output under varying inflation rates and nominal interest rates that satisfy the equilibrium conditions. Specifically, we first fix the nominal interest rate R and vary the inflation rates γ_c and γ_e from 1 to 10 percent. Then, we fix γ_e and vary R over the same range. The welfare levels derived from these exercises are reported in Figures 2 and 3.

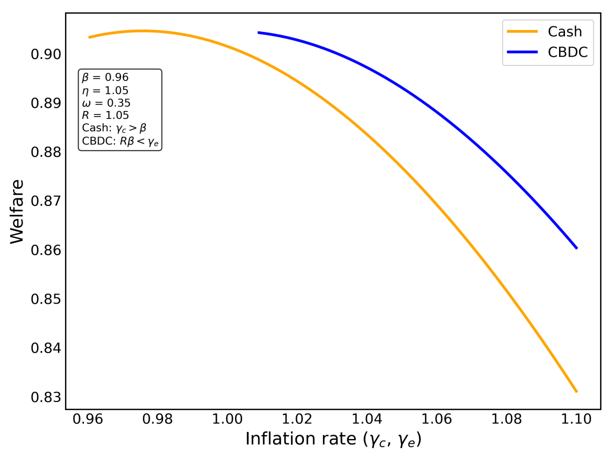

Figure 2: Welfare comparisons between cash and CBDC

The results indicate that the CBDC economy consistently delivers higher welfare than the cash economy over the range of equilibrium-consistent inflation rates. The welfare gap between CBDC and cash widens as inflation increases, but both economies experience diminishing welfare at higher inflation levels. This result suggests that although CBDC remains welfare-improving relative to cash, the overall welfare in both regimes declines as inflation rises. When examining the variation in the nominal interest rate, welfare in the CBDC economy increases monotonically with R within the equilibrium range, implying that higher nominal interest rates—consistent with the equilibrium restriction—are associated with improved welfare outcomes.

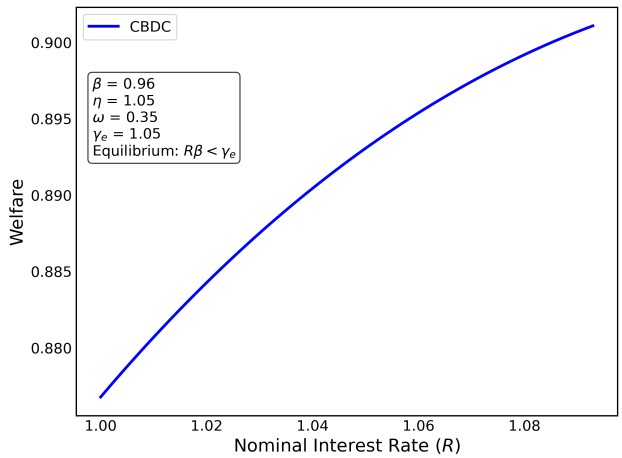

Figure 3: Welfare in a CBDC economy vs nominal interest rate

Figure 4 reports the results when key structural parameters are varied. The discount factor β has only a negligible negative effect on welfare gains up to the threshold beyond which the equilibrium no longer holds. The welfare gains from CBDC diminish as the preference-shock parameter η increases, while they rise monotonically with the production-function parameter ω. This result indicates that greater sensitivity of production to labor input amplifies the relative welfare advantage of CBDC, whereas stronger preference shocks reduce it. Overall, the equilibrium-constrained results confirm the robustness of the findings: CBDC adoption improves welfare relative to cash, with the welfare gap widening under moderate inflation before both economies experience diminishing returns at higher inflation levels.

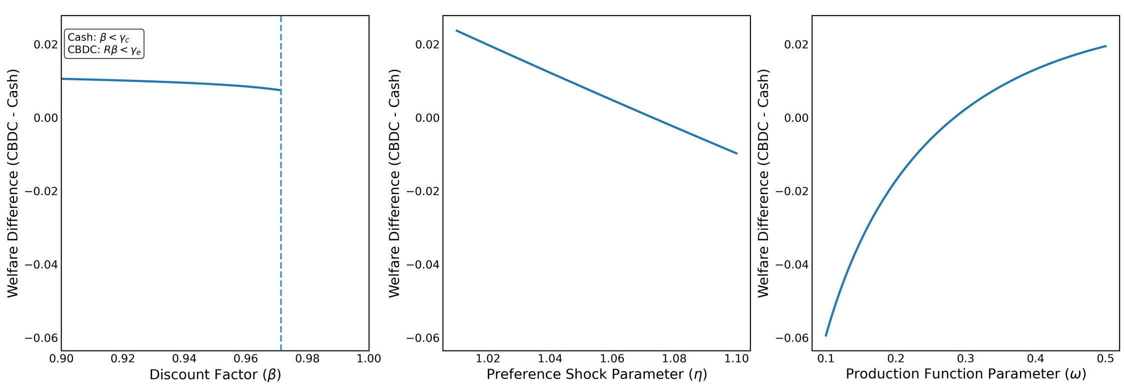

Figure 4: Welfare gains from CBDC as a function of key parameters

7 Conclusion

By introducing transaction transparency, CBDC enables governments to observe income and enforce taxation that is infeasible in cash-based systems. In our framework, labor taxes collected from CBDC users are non-distortionary, functioning as a mechanism that promotes discipline and allocative efficiency. Similarly, a voluntary CBDC fee can serve as a redistributive instrument, allowing the government to improve efficiency through targeted transfers.

Beyond addressing tax evasion, CBDC introduces redistributive channels unavailable in cash economies. Agents with higher marginal utility effectively subsidize the system through fixed access fees, while policymakers can adjust fees, inflation, and nominal interest rates to reallocate resources among heterogeneous agents. In impatient economies, welfare gains primarily arise through liquidity redistribution, whereas in patient economies, inflation differentials can influence saving behavior and the voluntary adoption of digital currency. Based on our findings, an optimal CBDC framework features higher nominal interest rates and lower inflation relative to cash regimes, promoting both efficiency and welfare improvements.

Our analysis abstracts from banking intermediation, partial privacy, and alternative welfare weights, yet the core insight remains robust. By curbing tax evasion enabled by cash anonymity, CBDC strengthens fiscal capacity while improving welfare through increased insurance against liquidity shocks and more efficient redistribution. As central banks advance the design of digital currencies, the trade-off between transaction privacy and fiscal capacity identified here is likely to remain central to future policy debates on CBDC design.

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